Seat No. : ____________

AD-111

April-2015

B.Sc., Sem.-VI

Mathematics

MAT-308 (Analysis-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) All the **five** questions are compulsory.

(2) Each question is of **14** marks.

1. (a) Prove :

(i) $f \in R[a, b] \Rightarrow U(f, P) - L(f, P) < \varepsilon$

OR

$U_p(f) - L_p(f) < \varepsilon$

where, P is any partition of [a, b].

(ii) $f, g \in R[a, b]$ then $f + g \in R[a, b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

OR

(a) Prove that if f is a continuous function on [a, b], then it is Riemann integrable on [a, b]. Is the statement $f \in R[a, b] \Rightarrow |f| \in R[a, b]$ true ? What about its converse ? Explain.

(b) Prove : If f is continuous on [a, b] and $F(x) = \int_a^x f(t)dt$ then $F'(x) = f(x)$ for all $a \le x \le b$.

Give suitable name to this result.

OR

(b) State the second mean values theorem for integrals. Is it possible to find $c \in (0, 1)$ such that the functions $f(x) = (1 + x^2)^{\frac{1}{2}}$ and $g(x) = 2x$ satisfy the second mean value theorem on [0, 1] ? Justify.

2. (a) State and prove the limit form of the comparison test for the convergence of the series. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n(n+1)}{3n^3 + 2n^2 + 4n - 7}$.

OR

(a) Prove the absolutely convergent series is convergent but the converse is not necessarily true. Discuss the absolute convergence of the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(\sqrt{n+2} - \sqrt{n-2})}{\sqrt{n} + 3}$$

(b) State Cauchy's condensation test and hence, prove that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges for $p > 1$ and it diverges for $p \le 1$ also, examine the convergence of the series $\sum_{n=3}^{\infty} \frac{1}{n \log n(\log(\log n))^2}$.

OR

(b) (i) If Σa_n is a divergent series of positive terms then show that the series $\Sigma \frac{a_n}{1 + na_n}$ is divergent.

(ii) If Σa_n is a convergent series of positive terms then show that the series $\Sigma \frac{\sqrt{a_n}}{n}$ is convergent.

3. (a) If the series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ converge absolutely to A and B respectively then prove that their Cauchy product series $\sum_{n=0}^{\infty} c_n$ is convergent and if C is the sum of Cauchy product then C = AB.

OR

(a) Define rearrangement of a series. If the series $\sum_{n=0}^{\infty} a_n$ is a series of non-negative terms converges to A then prove that every rearrangement of the series $\sum_{n=0}^{\infty} a_n$ converges to the same sum A.

(b) Define Power series centered at x_0. Discuss the convergence of the following power series stating interval of convergence :

(i) $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

(ii) $\sum_{n=0}^{\infty} n!\, x^n$

(iii) $\sum_{n=0}^{\infty} (-1)^n \frac{(x+1)^{2n}}{(n+1)2^n}$

OR

(b) Prove : If $\int_a^{\infty} f(x)\, dx$ converges absolutely, then $\int_a^{\infty} f(x)\, dx$ converges.

Test convergence :

(i) $\int_1^{\infty} \frac{1}{e^x + 1}\, dx$

(ii) $\int_{-\infty}^{0} \frac{dx}{4 + x^2}$

4. (a) State Taylor's theorem. Using Lagrangian form for the remainder, for any real x show that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots + \frac{x^n}{n!} + \ldots.$

OR

(a) For any real x, obtain Mac laurin series expansion of $\sin x$ hence, deduce the series for sin(1).

(b) Obtain the power series solution of the differential equation $(1 - x)y' - 2y = 0$ with the initial condition $y(0) = 4$.

OR

(b) State Binomial series theorem. Does this series converge to $(1 + x)^{\alpha}$ for $x \in [0, 1)$? Explain.

5. Attempt any **seven** :

(i) State the first mean value theorem for the R-integrable function.

(ii) Let a function f(x) = x, P = $\left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ be a partition of [0, 1] then compute U[f ; P] and L[f ; P].

(iii) Discuss the absolute convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.

(iv) State the Ratio test for the convergence of the series.

(v) Find the Cauchy product of $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ with itself.

(vi) Test the convergence of $\int_0^{\infty} \frac{1}{3^x} dx$

(vii) State the series of cos x, for any real x.

(viii) Examine the validity of the statement $ln\ 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$

(ix) If $y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$ and $y(x) = -2y'(x)$ then obtain relation among the coefficients.
